Dark Matter from Dual-Manifold Coupling

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Conventional approaches to dark matter typically require introducing new fields, particles, or interactions, or else modifying gravity. A recently proposed framework enlarges the Hilbert space of quantum theory by allowing a single quantum state to admit two conjugate dynamical descriptions—one on the familiar space–time manifold and one on a complementary momentum–energy manifold—thereby restoring a missing dynamical symmetry between the two. Here we show that dark matter arises naturally as a structural consequence of the coupling between these two dynamical sectors. When the coupling is non-boundary, analytic continuation in the momentum–energy manifold generates an evanescent projection onto the space–time manifold. The resulting configuration possesses defining properties of dark matter: it carries finite energy density and sources gravity, yet it remains non-propagating and produces no long-range electromagnetic radiation. These findings not only provide further support for the dual-manifold framework as a consistent extension of quantum theory, but also offer a new quantum-theoretic perspective on the origin of dark matter.

Dark matter remains one of the most profound and unresolved problems in modern cosmology. Its existence is inferred from a wide range of astronomical observations—such as galactic rotation curves, gravitational lensing, and cosmic microwave background anisotropies—all of which indicate that the majority of the universe's mass–energy content is invisible and nonluminous [1–3]. Despite decades of effort, the microscopic nature of dark matter remains unknown.

The challenge is twofold. From an astrophysical standpoint, dark matter is essential for explaining the formation and stability of galaxies and large-scale structures [4–7]. From a fundamental physics perspective, it represents a deep theoretical puzzle: it interacts gravitationally yet appears to evade all known interactions in the Standard Model [1–3]. Because of this unique status, dark matter has become a crucial testbed for new physics. Any fundamental theory aspiring to describe nature at its most basic level should be capable of explaining, or at least accommodating, dark matter in a natural and self-consistent way.

Recently, a new quantum framework—the dual-manifold theory—has been proposed to restore a conjugate dynamical symmetry between space–time and momentum–energy [8]. The universe is described by a single global quantum state with simultaneous projections onto two Fourier-conjugate manifolds, space-time manifold $\mathcal{M}_{xt}$ and momentum–energy manifold $\mathcal{M}_{kE}$. At the kinematic level, these manifolds are related by a norm-preserving Fourier isomorphism, but in the extended framework each manifold carries its own intrinsic unitary dynamics, making them distinct dynamical sectors of an enlarged Hilbert space. The two sectors are not isolated: consistency of the global state enforces boundary coupling between them. Remarkably, this structure reproduces the dark matter like background and exponential boundary mapping that underlies black-hole thermodynamics—normally derived only within general relativity.

In this work, we show that dark matter arises naturally from the dual-manifold framework. The key idea underlying the present analysis is that the two manifolds may be connected in more than one way. A boundary coupling corresponds to points where the singularity of one manifold maps onto the infinity of the other. In contrast, a non-boundary coupling allows the manifolds to connect through a finite analytic continuation in a complex neighborhood rather than at a single boundary point. We argue that such non-boundary coupling can lead to non-radiative, exponentially localized energy states that permeate the universe—naturally reproducing the essential phenomenology of dark matter. This result not only offers a new and powerful support for the dual-manifold framework itself, but also opens a conceptually new path toward understanding the origin of dark matter.

I. COUPLING OF THE DUAL MANIFOLDS

In the dual-manifold framework [8], the physical state of the universe is described by a single global quantum state living in an enlarged Hilbert space

$$\mathcal{H}_{\text{total}} = \mathcal{H}_{xt} \oplus \mathcal{H}_{kE},$$

where $\mathcal{H}_{xt}$ and $\mathcal{H}_{kE}$ are the dynamical sectors associated with the space–time manifold $\mathcal{M}_{xt}$ and the momentum–energy manifold $\mathcal{M}_{kE}$, respectively. At the kinematical level, the underlying L^2 function spaces are related by a norm-preserving Fourier isomorphism, but once autonomous dynamics is assigned to both sectors they are no longer unitarily equivalent as dynamical systems. Each sector carries its own intrinsic evolution,

$$i\hbar\partial_t\psi = H\psi, \qquad i\hbar\partial_E\phi = \tilde{H}\phi.$$

with $\psi(x,t) \in \mathcal{H}_{xt}$ and $\phi(k,E) \in \mathcal{H}_{kE}$.

In the absence of coupling, the only connection between the two manifolds is the standard unitary Fourier correspondence,

$$\psi(x,t) = \frac{1}{(2\pi\hbar)^2}\int e^{i(kx-Et/\hbar)}\phi(k,E)dkdE, \qquad (\equiv U\phi).$$

This transformation merely expresses representational equivalence: ψ and ϕ are two norm-preserving projections of the same global quantum state onto $\mathcal{M}_{xt}$ and $\mathcal{M}_{kE}$. With no coupling, the dual-manifold framework collapses to a purely mathematical correspondence and introduces no new physical structure—no interaction, no dynamical exchange, and no observable effects beyond standard quantum mechanics.

However, the two manifolds can be physically connected under certain conditions. Coupling becomes inevitable at the boundary where the singular limit of one manifold corresponds to the asymptotic infinity of the other. At such points the single global quantum state must satisfy both dynamical laws simultaneously, enforcing nontrivial boundary consistency. In general, this coupling can be expressed by introducing a weighting factor into the Fourier kernel:

$$K(x, t \mid k, E) = W(x, t; k, E)e^{i(kx-Et/\hbar)},$$

$$\psi(x, t) = \int K(x, t \mid k, E)\phi(k, E)dkdE.$$

When $W = 1$, the mapping reduces to the ordinary Fourier correspondence, and the two manifolds remain completely decoupled. When $W \neq 1$, it signifies a genuine physical coupling or information flow between the two manifolds.

The coupling kernel $W(x, t; k, E)$ can naturally be decomposed into an amplitude and a phase component,

$$W(x, t; k, E) = A(x, t; k, E)e^{i\Theta(x,t;k,E)}.$$

The amplitude A governs the spectral weight of the coupling, determining how strongly different regions of the momentum–energy manifold contribute to the projection, and thereby setting the large-scale density profile of the resulting state. The phase Θ encodes the geometric and topological structure of the connection: a nontrivial winding of $\Theta(k)$ generates residual coherence that prevents destructive interference, providing topological protection to the resulting evanescent states.

II. NON-BOUNDARY COUPLING AND THE EMERGENCE OF EVANESCENT STATES

We now examine the nature of the coupling between the two manifolds. In the dual-manifold framework, the singular limit of the space–time manifold corresponds to the asymptotic infinity of the momentum–energy manifold, and vice versa. These two extreme limits coincide under the dual correspondence, creating a single shared boundary point. At this boundary, the global quantum state must satisfy the dynamical constraints of both manifolds simultaneously, leading to what we call boundary coupling. In this case, information transfer occurs only through this isolated extremal limit—the two manifolds touch at precisely one point in their respective geometries.

Such coupling is necessarily localized and exceptional. It occurs only when one manifold is driven to its dynamical singular limit while the conjugate manifold reaches its asymptotic infinity. Black-hole horizons provide a concrete realization of this situation: each point on the event-horizon surface corresponds, under the dual mapping, to a direction of unbounded growth in momentum–energy. Thus the coupling is confined not to a single point, but to a codimension-one boundary surface defined by these mutually singular limits. These extreme circumstances represent only special regions of the universe and cannot describe generic inter-manifold behavior.

This raises a natural question: If the two manifolds coexist everywhere in the universe, why should their interaction be restricted to an isolated boundary point? In other words, is it possible for the manifolds to communicate in a smoother, more extended fashion, without requiring singular or asymptotic limits?

To describe such broader interaction, we introduce non-boundary coupling. Here the correspondence between the two manifolds is extended from the singular boundary surface into a small neighborhood around it. This requires an analytic continuation of the boundary mapping: instead of meeting only at the real singular limits, the global state is continued into a thin complexified region around the boundary. In this continuation, the spectral variables acquire a small imaginary component, allowing the state to pass smoothly through the collar region and reemerge on the other manifold. This construction replaces isolated boundary contact with a controlled overlap region, enabling a more general and physically distributed form of coupling.

Concretely, consider a momentum–energy–manifold mode of the form e^{ikx} (we suppress t for clarity). When this mode traverses the complex collar to enter the x–t manifold, its wavenumber continues to

$$k \longrightarrow k_r + i\kappa, \qquad \kappa > 0.$$

The real part k_r encodes ordinary oscillation, while the imaginary part κ encodes exponential decay in the direction normal to the boundary traversal. A minimal analytically controllable spectral profile is the Yukawa form centered at a real carrier wavenumber k_r,

$$\phi(k) = \frac{1}{(k - k_0)^2 + \kappa_0^2}, \qquad \kappa_0 > 0,$$

which is analytic on the real axis and extends smoothly into the complex domain through simple poles at

$$k = k_0 \pm i\kappa_0.$$

Extending k from the real to the complex domain already leads to a significant effect. To see this, let us first take $W = 1$ and use the Fourier map without additional weighting,

$$\psi(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \phi(k)\mathrm{e}^{ikx}dk.$$

With the Yukawa choice above, residue evaluation in the upper or lower half-plane yields

$$\psi(x) = \frac{1}{2\kappa_0} e^{-\kappa_0|x|} e^{ik_0 x},$$

an exponentially localized envelope with an in-envelope carrier oscillation.

This corresponds to an evanescent state on $\mathcal{M}_{xt}$: its amplitude is exponentially localized and the probability current,

$$J = \frac{\hbar}{2mi}(\psi^* \nabla \psi - \psi \nabla \psi^*) = \frac{\hbar k_0}{m} |\psi|^2 = \frac{\hbar k_0}{4m\kappa_0^2} e^{-2\kappa_0|x|},$$

decays exponentially with $|x|$. Its divergence,

$$\nabla \cdot J = -\frac{\hbar k_0}{2m\kappa_0} \operatorname{sgn}(x) e^{-2\kappa_0|x|} + \frac{\hbar k_0}{2m\kappa_0^2} \delta(x),$$

is nonzero, indicating that the mode cannot sustain a steady flux or a well-defined group velocity: it is confined to a finite region and cannot propagate as a freely traveling wave.

III. NON-BOUNDARY COUPLING KERNEL AND EMERGENT RADIAL SCALING

Having examined the effect of complex continuation with a single fixed decay rate, we now treat the non-boundary coupling properly by introducing a nontrivial coupling kernel,

$$W(\mathbf{r}, \mathbf{k}) = A(\kappa) e^{i\Theta(\mathbf{k})}.$$

Instead of a single decay rate κ_0, we now allow a continuous range of decay parameters $\kappa > 0$, so that an entire band of evanescent modes can contribute coherently. The amplitude $A(\kappa)$ defines the spectral weight assigned to each decaying component. We adopt a smooth, normalizable form that produces a continuous decay spectrum,

$$A(\kappa) = C\sqrt{\kappa} e^{-\kappa/\kappa_0}, \qquad \kappa > 0,$$

where κ_0 sets the characteristic low-κ (large-scale) cutoff, and C is an overall normalization or cosmological scaling constant.

The coupling kernel acquires a topological phase as a direct consequence of analytic continuation. To define a non-boundary coupling, the momentum variable is promoted to the complex plane, $k \in \mathbb{C}$, and the integration contour is deformed away from the real axis so as to bypass the singular points of the boundary map. The coupling kernel $W(k)$ remains analytic and nonvanishing along the deformed contour, but the contour now encloses the isolated poles or branch points inherited from the boundary Green's function.

By the argument principle, the phase of $W(k)$ must undergo an integer multiple of 2π as the contour loops around such singularities:

$$m = \frac{1}{2\pi} \oint d\Theta = \frac{1}{2\pi i} \oint d \log W \in \mathbb{Z},$$

Locally, this forces the phase to take the universal vortex-like form

$$\Theta(k) \simeq m \arg(k - k_0), \qquad m \in \mathbb{Z},$$

up to a smooth analytic contribution. This winding is therefore not an arbitrary choice but a topological imprint of the analytic continuation path around the boundary singularity. The factor $e^{i\Theta(\mathbf{k})}$ breaks perfect angular isotropy in the momentum distribution, in direct analogy with optical vortices or vortex phases in superconductivity.

A. Radial projection and averaged density

To project the coupled state from $\mathcal{M}_{kE}$ onto $\mathcal{M}_{xt}$, we start from

$$\psi(\mathbf{r}) = \int d^3k W(\mathbf{k}) \phi(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}, \qquad W(\mathbf{k}) = A(\kappa) e^{i\Theta(\mathbf{k})}.$$

During the angular integration, the winding number associated with the the phase factor $e^{i\Theta(k)}$ enforces a selection rule on the angular Fourier components of $\phi(\mathbf{k})$: the $e^{im\theta_k}$ factor shifts each mode $e^{in\theta_k}$ to $e^{i(n+m)\theta_k}$, so that only the $n = -m$ component survives the angular average. This produces a nonvanishing residual coherence factor C_m, which prevents the extended modes from cancelling completely. The coherence factor is absorbed into the overall normalization, leaving the radial profile governed solely by the amplitude spectrum $A(\kappa)$. Thus, the radial wavefunction can be expressed as a superposition of elementary components, each characterized by a single decay constant κ:

$$\psi(r) \propto \int_0^\infty A(\kappa) f_\kappa(r) d\kappa, \qquad r = |\mathbf{r}|.$$

For each fixed κ, we use evanescent kernel

$$f_\kappa(r) = e^{-\kappa r},$$

which corresponds to the physical, non-divergent solution that defines an evanescent mode. Substituting this kernel gives

$$\psi(r) \propto \int_0^\infty A(\kappa) e^{-\kappa r} d\kappa \sim \int_0^\infty \sqrt{\kappa} e^{-\kappa/\kappa_0} e^{-\kappa r} d\kappa.$$

The observable (or gravitationally effective) energy density is obtained by taking the squared modulus and averaging over fast phase fluctuations:

$$\rho(r) \equiv \overline{|\psi(r)|^2} \propto \int_0^\infty \kappa e^{-2\kappa/\kappa_0} e^{-2\kappa r} d\kappa.$$

Combining the exponents and defining

$$a(r) \equiv 2r + \frac{2}{\kappa_0}, \qquad a(r) > 0,$$

the remaining Laplace-type integral is elementary:

$$\int_0^\infty \kappa e^{-a(r)\kappa} d\kappa = \frac{1}{a(r)^2}.$$

Hence we obtain a closed analytic form for the radial energy-density profile (absorbing constants into ρ_0):

$$\rho(r) = \frac{\rho_0}{\left(r + 1/\kappa_0\right)^2}.$$

B. Correspondence with dark matter

Thus the state obtained from the non-boundary dual-manifold coupling naturally satisfies the physical requirements expected of dark matter [1].

First, the projection onto $\mathcal{M}_{xt}$ yields a stationary energy density $\rho(r) = |\psi(r)|^2$, which sources gravity in the usual way. No additional assumptions about the interaction are required: any nonzero real energy density contributes to the stress–energy tensor.

Second, the state is intrinsically non-propagating. Each elementary component $e^{-\kappa r}$ is evanescent and carries no outgoing flux or radiation. After spectral superposition, the full configuration remains non-radiative and produces no far-field electromagnetic signal. Importantly, the dual-manifold framework does not impose any assumption about whether the underlying degrees of freedom carry electric charge. If the underlying constituents do possess a charge degree of freedom, the non-propagating nature of the evanescent state implies that only localized electromagnetic interactions could occur, confined to regions where the mode has support. Such interactions would not produce propagating electromagnetic waves and therefore would not conflict with observational bounds on long-range electromagnetic emission. Therefore, the absence of outward radiation is guaranteed by the evanescent structure of the state rather than by any ad-hoc restriction on its charge content.

Taken together, these properties—gravitational mass and radiative darkness—are precisely the generic criteria required for a viable dark-matter distribution. The detailed radial profile depends on the chosen spectral amplitude $A(\kappa)$ and momentum-space wavefunction $\phi(k)$, but the dark-matter–like behavior itself is universal to the non-boundary coupling mechanism in the dual-manifold framework.

C. Three-regime radial scaling

Although the detailed radial profile depends on the chosen spectral amplitude $A(\kappa)$ and wavefunction $\phi(k)$, it is instructive to compare the profile obtained from our minimal spectral model with observational dark-matter halos.

In the mathematical construction, r denotes the spatial coordinate on $\mathcal{M}_{xt}$ and carries no fixed physical scale. When the wavefunction amplitude $|\psi(r)|^2$ is interpreted as an effective energy density, r can be rescaled to represent the physical radial distance in galactic or cosmological units:

$$\rho(r) = \frac{\rho_0}{(r + 1/\kappa_0)^2} \quad \longrightarrow \quad \rho_{\rm phys}(r) = \frac{\rho_0'}{(r + r_c)^2},$$

where $r_c = \lambda_r/\kappa_0$ defines the characteristic core radius of the galactic dark-matter halo. This rescaling is not arbitrary: it corresponds to expressing the evanescent decay length $1/\kappa_0$ in macroscopic units. The rescaled profile provides a natural basis for analyzing three distinct radial regimes—core, transition, and outer tail—that correspond closely to the dark-matter density distributions observed in galaxies.

1. Core region ($r \ll r_c$): The density approaches a finite constant, forming a smooth, nonsingular central core. Unlike empirical halo models such as the Navarro–Frenk–White profile [9, 10], which diverge as $\rho \sim 1/r$ near the galactic center and produce an unphysical cusp, the present result remains finite and naturally yields a cored structure.

2. Intermediate region ($r_c \ll r \ll r_{\rm out}$): The density follows a power law, $\bar{\rho}(r) \propto 1/r^2$, leading to two key astrophysical consequences. First, the enclosed mass scales linearly with radius, $M(r) \sim \int_0^r \rho(r')r'^2 dr' \propto r$, so the circular velocity $v_c = \sqrt{GM(r)/r}$ remains nearly constant—reproducing the flat rotation curves that originally motivated the dark-matter hypothesis [11]. Second, the $1/r^2$ scaling extends the gravitational potential well beyond the visible disk, providing sufficient mass to bind the outer stellar orbits. Thus, this regime embodies the essential astrophysical role of dark matter: a large-scale, non-radiative mass distribution that maintains galactic coherence without luminous emission.

3. Outer region ($r \gtrsim r_{\rm out}$): At sufficiently large radii, the behavior of the density profile is controlled by the high-κ portion of the spectrum $A(\kappa)$. In the minimal model used above, $A(\kappa) \propto \sqrt{\kappa} e^{-\kappa/\kappa_0}$, the integral produces a long-range $1/r^2$ tail with no intrinsic cutoff. In more realistic settings, however, the spectral amplitude need not extend indefinitely. If the dark-sector microphysics imposes a high-κ suppression (for example through a natural UV cutoff or a finite analytic continuation domain), the profile acquires an exponential outer decay,

$$\rho(r) \sim e^{-2r/r_{\rm out}},$$

which regularizes the total mass and yields a finite halo extent.

IV. TEMPORAL STRUCTURE FROM ENERGY–TIME ANALYTIC CONTINUATION

In the analysis above we focused on the spatial structure of the coupled state, effectively working in a stationary limit where the explicit dependence on the energy–time pair (E, t) was suppressed. This is appropriate for describing present–day dark–matter halos, where observations probe the spatial mass distribution at a given cosmological epoch. It is natural to ask how the non–boundary coupling manifests itself in time once the energy degree of freedom is restored.

The construction is completely parallel to the spatial case. In the momentum–energy manifold $\mathcal{M}_{kE}$, non–boundary coupling requires analytic continuation of the energy variable across the interface between the two manifolds,

$$E \longrightarrow E_r - i\varepsilon, \qquad \varepsilon > 0,$$

so that each spectral component acquires a decaying temporal envelope. When this continued mode is projected onto the space–time manifold $\mathcal{M}_{xt}$, the temporal factor takes the form

$$e^{-iEt/\hbar} = e^{-iE_r t/\hbar} e^{-\varepsilon t/\hbar},$$

where E_r governs phase rotation and ε controls exponential decay in time. After averaging over the fast oscillatory phase, the slowly varying temporal envelope of the coupled state can be written as a superposition over decay rates:

$$\psi_{\rm env}(t) \propto \int_0^\infty B(\varepsilon) e^{-\varepsilon t/\hbar} d\varepsilon,$$

with $B(\varepsilon)$ the effective spectral weight generated by the dynamics on $\mathcal{M}_{kE}$ and by the coupling kernel W.

This expression is the direct temporal analogue of the spatial superposition

$$\psi(r) \propto \int_0^\infty A(\kappa) e^{-\kappa r} d\kappa.$$

Each individual component $e^{-\varepsilon t/\hbar}$ represents a transient evanescent mode in time, but the full temporal behavior of the dark configuration is determined by the choice of the spectral distribution $B(\varepsilon)$. Depending on the form of $B(\varepsilon)$, the resulting energy density $\rho(r, t) \sim |\psi(r, t)|^2$ can describe configurations that are nearly stationary on cosmological timescales, slowly evolving halos, or more transient structures. In particular, if $B(\varepsilon)$ is strongly peaked near $\varepsilon = 0$, the temporal envelope varies only weakly over a Hubble time, consistent with the empirical success of quasi–static halo models, while still allowing for a controlled long–time evolution within the same framework.

A detailed confrontation between the temporal evolution implied by a specific choice of $B(\varepsilon)$ and cosmological data lies beyond the scope of the present work. The key point here is structural: once energy is allowed to evolve dynamically and analytic continuation is applied to the (t, E) sector, the dual–manifold framework naturally generates a time–dependent generalization of the evanescent dark state, with its precise time dependence controlled by the underlying spectral function rather than fixed ad hoc.

V. CONCLUDING REMARKS

Existing dark-matter models typically fall into two broad categories: (1) particle-based scenarios—such as WIMPs [2, 12], axions [13], sterile neutrinos [14], or dark photons [15]—which introduce new fields and interactions, and (2) modifications of gravity designed to account for galactic rotation curves without invoking non-luminous matter [16, 17]. The framework developed here differs fundamentally from both. Dark matter in the dual-manifold theory is not a new particle species, nor does it require altering gravitational laws; instead, it arises as an evanescent projection of a single quantum state across two dynamically conjugate manifolds.

The deeper insight is structural. In standard quantum mechanics, dynamical evolution is restricted to the time variable, while the energy coordinate is kinematical and admits no autonomous dynamics, analytic continuation, or boundary structure. As a result, the conventional framework contains no intrinsic mechanism for generating non-luminous gravitational matter; constructing dark matter requires adding new degrees of freedom or modifying gravity by hand. This foundational limitation explains why dark-matter model building within the standard paradigm is inherently difficult.

By contrast, once the missing dynamical symmetry between time and energy is restored, the quantum description acquires a second autonomous evolution sector in the momentum–energy manifold [8]. A single global state evolves simultaneously under two independent generators, and consistency across the interface between manifolds forces the wave to traverse a complexified collar when the coupling is non-boundary. Analytic continuation is then unavoidable, introducing imaginary momentum components that yield evanescent, non-propagating projections on the space–time manifold. These projections possess finite gravitational energy density yet produce no far-field electromagnetic radiation. Importantly, these properties are not imposed phenomenologically; they arise directly and inevitably from the enlarged dynamical structure of the dual-manifold theory.

In this way, the emergence of dark-matter–like states is a structural consequence of completing a foundational symmetry of quantum dynamics, rather than extending the Standard Model. The results presented here therefore strengthen the dual-manifold framework as a coherent quantum–theoretic foundation and offer a new physical perspective on the origin of dark matter.

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